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FLUID FLOW IN A CURVED PIPE WITH MAGNETIC FIELD ALONG THE CENTRE LINE

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ABSTRACT
Numerical study is performed to investigate the Magnetohydrodynamic fluid flow through a curved pipe with circular cross-section under various conditions to examine the combined effects of high Dean number D_n, magnetic parameter M_g and curvature δ. Spectral method is applied as a main tool for the numerical technique; where, Fourier series, Chebyshev polynomials, Collocation methods, and Iteration technique are used as secondary tools. The flow patterns have been shown graphically for large Dean numbers as well as large magnetic parameters and a wide range of curvatures $0.01 \le \delta \le 0.9$.Two vortex solutions have been found. Axial velocity has been found to increase with the increase of Dean number and decreases with the increase of curvature and magnetic parameters. For high magnetic parameter and Dean number and low curvature the axial flow shifted towards the centre of the pipe as a result almost all the fluid particles strength are week.

Keywords: Dean Number, Magnetic Parameter and Curvature

1. INTRODUCTION

The flow through a curved duct driven by a pressure gradient force has been studied considerably because of its practical importance in chemical, mechanical and biological engineering. Curved ducts are used as parts of pipe line, heat exchangers, cooling system, chemical reactors, gas turbines, centrifugal pumps etc. Used of curved ducts are also found in human arterial system. Such type of flow is called Dean Flow. Dean [2,3] was the first author who formulated the problem in mathematical terms under fully developed flow conditions.

For a circular tube Dennis and Ng [4] found the dual solutions using Fourier finite difference method. In the same year Nandakumar and Mashiyah [8] found the phenomena but they used Finite Difference Method. Yanase *et al.* [13] studied analysis for flow in a curved duct with circular cross-section.

Magnetohydrodynamics (MHD) is the academic discipline which studies the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water.

A number of magnetic confinement fusion reactor (MCFR) concepts have been based on the use of liquid lithium or lithium lead for the dual function of tritium breeding and cooling of first wall/blanket structures. In these “self-cooled” concepts, the conducting fluid flows in a strong magnetic field. The applied magnetic field, which is primarily intended for a plasma confinement, introduces significant body forces that can drastically influence fluid motion. If the imposed magnetic field is parallel to the flow, no magnetic body force arises.

However, when the imposed magnetic field is transverse to the flow, the effect of induced magnetic body forces must be considered.

Flow in curved pipe with no magnetic field is reviewed by Berger *et al.*[10] As early as 1928, Dean [11] presented an analytical series solution to the fully developed flow of non-conducting fluids in curved pipes of small curvatures. The Navier-Stokes equations in curved pipes have been solved numerically by McConalogue and Srivastava [5] for intermediate Dean numbers and by Collins and Dennis [12] for high Dean numbers.

The effects of the magnetic field on fluid flow have been studied primarily for straight pipes [1, 6, 7, and 9]. Shercliff [6] solved the problem of flow in circular pipes under transerves magnetic fields in an approximate manner foe large Hartmann numbers assuming walls of zero and small conductivity. The effect of wall conductivity was also studied by Chang and Lundgren [1] . Pressure drop in thin walled circular straight ducts was studied by Holroyed and Walker [9] , neglecting the inertial effects and induced magnetic field. Recently, Walker [7] developed solutions to MHD flow equations by asymptotic analysis for circular straight ducts under strong transverse magnetic fields.

Hence, our aim is to obtain a detail results on the Dean numbers as well as magnetic parameter and a wide range of curvatures $0.01 \le \delta \le 0.9$. In this present study, the magnetic field has been imposed along the centre line of a curved pipe.

2. PROBLEM FORMULATION

For the curved pipe magnetic flow the coordinate systems (r, α, θ) as shown in the Figure 1. where, O is the centre of curvature, L is the radius of the pipe, a is the radius of the cross-section, α is the circumferential angle, θ is the axial variable and r is the radial variable.

2.1 Governing Equation

The basic equations for steady-state laminar flow are Continuity equation becomes:

$$\nabla . \boldsymbol{q} = 0 \tag{1}$$

Momentum equation becomes:

$$\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{t}} + (\boldsymbol{q}.\nabla)\boldsymbol{q} = -\frac{1}{\rho}\nabla \boldsymbol{p} + \nu \nabla^2 \boldsymbol{q} + \frac{1}{\rho} \mathrm{J} \wedge \mathrm{B} \tag{2}$$

where **J** is current density and **B** is the magnetic induction.

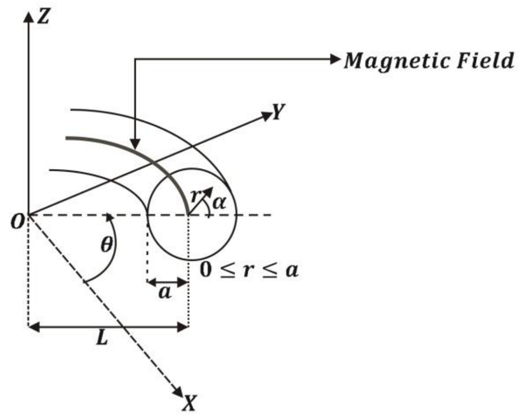

Fig 1. Toroidal Coordinate system for curved pipe with magnetic field

2.2 Toroidal Coordinate System

Let us defined the following nondimensional variables:

$$u' = \frac{q_r}{\frac{\upsilon}{a}};\; v' = \frac{q_\alpha}{\frac{\upsilon}{a}};\; w' = \frac{q_\theta}{\frac{\upsilon}{a}}\sqrt{\frac{2a}{L}};\; r' = \frac{r}{a}$$

$$S' = \frac{L\theta}{a};\; \frac{a}{L} = \delta;\; p' = \frac{p}{\rho\left(\frac{\upsilon}{a}\right)^2}$$

where u', v', w' are non-dimensional velocities along the radial, circumferential and axial direction respectively. r' is non-dimensional radius, S' is the nondimensional axial variable, δ is non-dimensional curvature and p' non-dimensional pressure.

Non-dimensional continuity equation:

$$r'\left(L + ar'\cos\alpha\right)\frac{\partial u'}{a\partial r'} + \frac{\left(L + ar'\cos\alpha\right)u'}{a} + r'\left(u'\cos\alpha - v'\sin\alpha\right) + r'\sqrt{\frac{L}{2a}}\frac{\partial w'}{\partial \theta} = 0 \tag{3}$$

Non-dimensional radial momentum equation:

$$u'\frac{\partial u'}{\partial r'} + \frac{v'}{r'}\frac{\partial u'}{\partial \alpha} - \frac{v'^2}{r'} - \frac{Lw'^2\cos\alpha}{2\left(L + ar'\cos\alpha\right)} = -\frac{\partial p'}{\partial r'} - \frac{\partial}{r'\partial\alpha}\left(\frac{\partial v'}{\partial r'} + \frac{v'}{r'} - \frac{\partial u'}{r'\partial\alpha}\right) - \sigma'\mu_e a^2 u' H_\theta^2 \tag{4}$$

Non-dimensional circumferential momentum equation:

$$r'u'\frac{\partial v'}{\partial r'} + \frac{v'}{r'}\frac{\partial v'}{\partial\alpha} + u'v' + \frac{Lr'\sin\alpha}{2\left(L + ar'\cos\alpha\right)}w'^2 = -\frac{\partial p'}{\partial\alpha} + r'\frac{\partial}{\partial r'}\left(\frac{\partial v'}{\partial r'} + \frac{v'}{r'} - \frac{\partial u'}{r'\partial\alpha}\right) - \sigma'\mu_e a^2 v' H_\theta^2 \tag{5}$$

Non-dimensional axial momentum equation:

$$\begin{aligned}
&\left(u'\frac{\partial}{\partial r'} + \frac{v'}{r'}\frac{\partial}{\partial\alpha}\right)w' + \frac{a\cos\alpha}{L + ar'\cos\alpha}u'w' - \frac{a\sin\alpha}{L + ar'\cos\alpha}v'w' \\
&= -\frac{1}{L + ar'\cos\alpha}\frac{a^3}{\rho\upsilon^2}\sqrt{\frac{2a}{L}}\frac{\partial p}{\partial\theta} \\
&+ \left\{\begin{array}{l}\left(\frac{1}{r'} + \frac{\partial}{\partial r'}\right)\frac{\partial w'}{\partial r'} + \left(\frac{1}{r'} + \frac{\partial}{\partial r'}\right)\frac{aw'\cos\alpha}{L + ar'\cos\alpha} \\ + \frac{1}{r'^2}\frac{\partial^2 w'}{\partial\alpha^2} - \frac{a\partial}{r'\partial\alpha}\left\{\frac{w'\sin\alpha}{L + ar'\cos\alpha}\right\}\end{array}\right\}
\end{aligned} \tag{6}$$

The other variables without primes are dimensional variables. Constant pressure gradient force is applied along the axial direction through the centre of cross section. At the centre of the cross-section $r = 0$ and at the boundary of the cross-section $r = a$, where all the velocity components are zero. In dimensionless form this reduces to $r' = 0$ at the centre of cross-section and $r' = 1$ at the boundary of the cross-section. With the help of the above dimensionless variables and the boundary conditions the equation of motion reduces to the following form:

$$\frac{1}{r'}\left\{\frac{\partial\psi}{\partial r'}\frac{\partial(\Delta\psi)}{\partial\alpha} - \frac{\partial\psi}{\partial\alpha}\frac{\partial(\Delta\psi)}{\partial r'}\right\} + \Delta^2\psi + w'\left(\sin\alpha\frac{\partial w'}{\partial r'} + \frac{\cos\alpha}{r'}\frac{\partial w'}{\partial\alpha}\right) - M\Delta\psi = 0 \tag{7}$$

and $\frac{1}{r'}\left(\frac{\partial\psi}{\partial r'}\frac{\partial w'}{\partial\alpha}-\frac{\partial\psi}{\partial\alpha}\frac{\partial w'}{\partial r'}\right)+\Delta w'+D_n=0$ (8)

Equation (7) and (8) are called secondary and axial flow respectively. where, $\Delta\equiv\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}+\frac{1}{r'^2}\frac{\partial^2}{\partial\alpha^2}$, $G=-\frac{\partial p}{\partial S}$ and $D_n=\frac{a^3}{\mu\upsilon}\sqrt{\frac{2a}{L}}G$, $M=\sigma'\mu_e a^2 H_\theta^2$. Here, ψ is the stream function defined by, $u'=\frac{1}{r'}\frac{\partial\psi}{\partial\alpha}$, $v'=-\frac{\partial\psi}{\partial r'}$, G is the constant pressure gradient force, μ is the viscosity, υ is the kinematic viscosity, D_n is the Dean number and M_g is the magnetic parameter. The dimensionless flux κ is given by, $\kappa=\frac{\sqrt{2}}{\pi}\int_0^1 r'\int_0^{2\pi} w'd\alpha dr'$

3. NUMERICAL TECHNIQUE

The theoretical treatments of flow in a curved pipe have been made either analytically or numerically. The present work is mainly based on numerical methods. For this purpose the Spectral method has been used to solve the equations (7) and (8). As for the spectral collocation method, which will be mainly used in this paper, it is necessary to discuss the method in brief. The expansion by polynomial functions is utilized to obtain steady or non-steady solution. Fourier series and Chebyshev polynomials are used in circumferential and radial directions respectively. Assuming that steady solution is symmetric with respect to the horizontal line of the cross-section, ψ and w' are expanded as,

$$\psi(r',\alpha)=\sum_{n=1}^{N} f_n^s(r')\sin n\alpha+\sum_{n=0}^{N} f_n^c(r')\cos n\alpha$$

and $$w'(r',\alpha)=\sum_{n=1}^{N} w_n^s(r')\sin n\alpha+\sum_{n=0}^{N} w_n^c(r')\cos n\alpha$$

The collocation points are taken to be, $R=\cos\left\{\frac{N+2-i}{N+2}\right\}\pi$ [$1\le i\le N+1$]. Then we get non-linear equations for $W_{nm}^s, W_{nm}^c, F_{nm}^s, F_{nm}^c$. The obtained non-linear algebraic equations are solved under by an iteration method with under-relaxation. Convergence of this solution is taken up to five decimal places by taking $\varepsilon_p<10^{-5}$. Here, p is the iteration number. The values of M and N are taken to be 60 and 35 respectively for better accuracy. Where, N is the truncation number of the Fourier series. where,

$$\varepsilon_p=\sum_{n=1}^{N}\sum_{m=0}^{M}\left[\begin{array}{l}\left(F_{mn}^{s(p)}-F_{mn}^{s(p+1)}\right)^2\\+\left(W_{mn}^{s(p)}-W_{mn}^{s(p+1)}\right)^2\end{array}\right]+\sum_{n=0}^{N}\sum_{m=0}^{M}\left[\begin{array}{l}\left(F_{mn}^{c(p)}-F_{mn}^{c(p+1)}\right)^2\\+\left(W_{mn}^{c(p)}-W_{mn}^{c(p+1)}\right)^2\end{array}\right]$$

4. RESULT AND DISCUSSION

Flow through a curved pipe of circular cross section with Magnetic parameter has been considered. The flow is governed by two non dimensional parameters: the Dean number (D_n) and the magnetic parameter (M_g). In this paper, steady laminar flow for viscous incompressible fluid has been analyzed under the action large Dean numbers as well as magnetic parameter at curvatures $\delta=0.1$. The flow pattern have been shown in contour plots of the axial velocity, stream lines and vector plots of secondary flow in each figure.

Variation of Flux (κ) with the variation of magnetic parameter (M_g) has been shown in figure 2, figure 3 figure 4 and figure 5. for different Dean number (D_n). And it is clear that the flux increases with the increase of magnetic parameter. But if the magnetic parameter increases continuously the rate of change of flux is negligible. For each figure the total flow is found to decreases as the magnetic parameter increase. The stream line, vector plots of the secondary flows and the contour plots of axial velocity in a circular cross section for $M_g=500, 1000, 1500, 2000, 3000$ at curvature $\delta=0.1$ have been shown in figure 2, figure 3, figure 4 and figure 5.

For each figure the outer wall is to the right and the inner wall is to the left. The length of arrow indicates the ratio of the stream velocity to the axial velocity and the direction of the flow in vector plots are always indicates by an arrowhead, no matter how small the flow is. Thus, the relative strength of the flow is not resolved for areas of a very weak secondary flow. In figure 3. the vector plots of the secondary flow show the direction of the fluid particles and the strength of the vortex is shifted towards outer half of the cross-section as magnetic parameter increases.

In case of secondary flow behavior, symmetric contour plots have been found which are shown in figure. 2, figure 3, figure 4 and figure 5. As magnetic parameter increases there originate a secondary flow and only 2-vortex solution has been found for the secondary flow. The two vortexes are of same strength but rotating in counter clockwise direction.

In figure 4, the axial flow is greater in magnitude than secondary flow and it varies a great deal with magnetic parameter. As a result the difference between two consecutive contours line of the axial flow have been taken different for different magnetic parameters and different Dean numbers, which are given in the table 1.

Table 1: Difference between the contours for various values of Dean number and Magnetic parameter

D_n	Magnetic parameter M_g				
	500	1000	1500	2000	3000
600	0.035	0.020	0.015	0.015	0.015
800	0.055	0.035	0.035	0.020	0.015
1500	0.15	0.10	0.10	0.06	0.05
1000	Magnetic parameter M_g				
	1000	2000	3000	4000	5000
	0.045	0.030	0.020	0.020	0.015

In case of axial flow behavior, the axial flow is also symmetric. The fluid particles are shifted towards the outer wall of the cross section and form a *low velocity band* inside the outer wall of the cross-section in figure 4. As magnetic parameter decreases the magnitude of the axial flow gets higher. The axial flow decreases with the increase of Dean number. The maximum flux is found for the highest magnetic parameter $M_g = 3000$ and

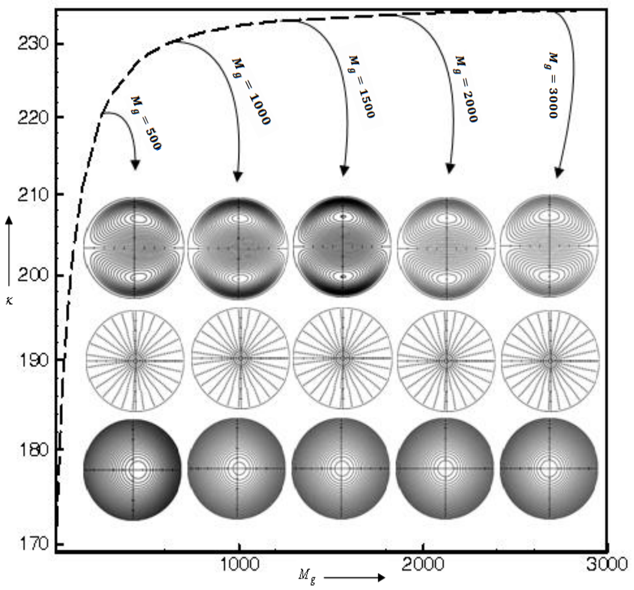

Fig 2. Flux κ versus magnetic parameter M_g for Dean number $D_n = 600$ and Stream lines, vector plots of secondary flow and contour plots of axial velocity for different values of Magnetic parameter.

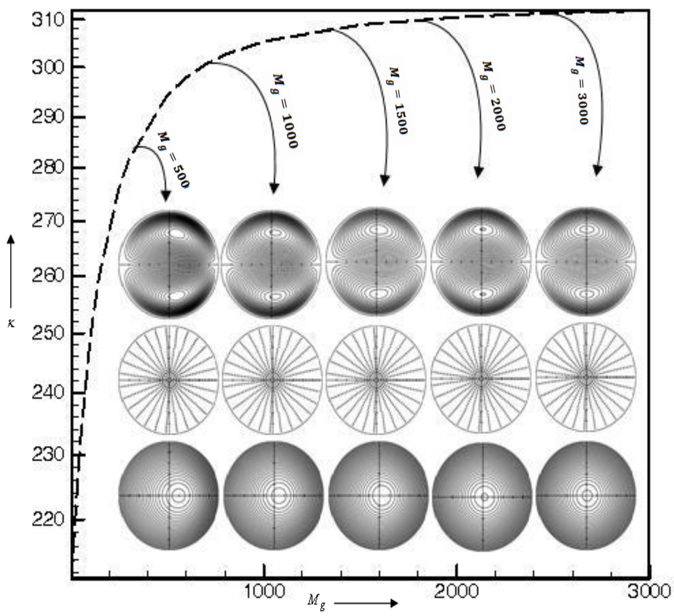

Fig 3. Flux κ versus magnetic parameter M_g for Dean number $D_n = 800$ and Stream lines, vector plots of secondary flow and contour plots of axial velocity for different values of Magnetic parameter.

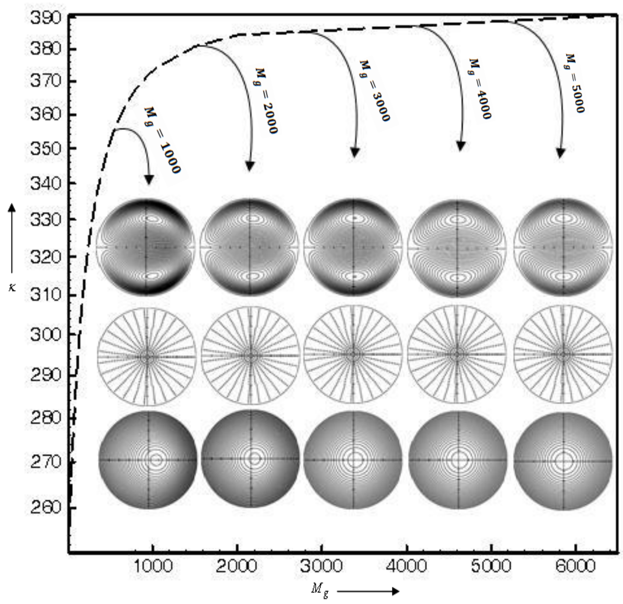

Fig 4. Flux κ versus magnetic parameter M_g for Dean number $D_n = 1000$ and Stream lines, vector plots of secondary flow and contour plots of axial velocity for different values of Magnetic parameter.

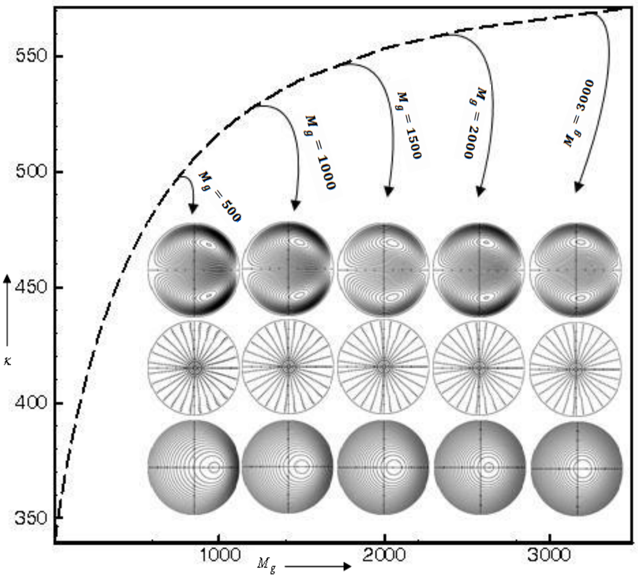

Fig 5. Flux κ versus magnetic parameter M_g for Dean number $D_n = 1500$ Stream lines, vector plots of secondary flow and contour plots of axial velocity for different values of Magnetic Parameter.

and in this case contour plots of the axial flow reveals that almost all the particles have been shifted towards the outer half of the cross-section in figure. 2.

In figure 3. figure 4. and figure 5. the contours are nearly shifted circular and are eccentric with their centre shifted towards the outer wall of the tube.

The effect of continuous change of curvature on the flow will be shown in our further study.

5. CONCLUSION

From the result the following conclusion can be drawn:

1. Two vortex solutions have been found.
2. The strength of the vortices is shifted to the outer half from the inner half with the increase of magnetic parameter.
3. For high magnetic parameter, Dean number and low curvature, the axial flow is shifted towards the centre of the pipe as a result almost all the fluid particles strength are week.

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7. NOMENCLATURE

Symbol	Meaning
δ	Curvature
M_g	Magnetic Parameter
D_n	Dean number

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